

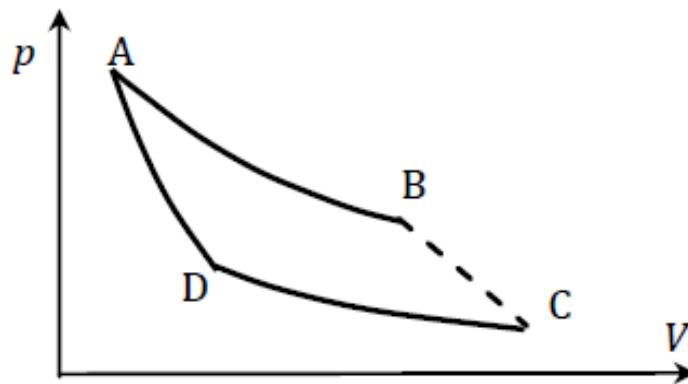
Version 1 and version 3

PHYSICS 1 – A.Y. 2024/2025 Session 1 – JUNE 27, 2025

PROBLEM 3: Thermodynamics [8 points]

One mole of diatomic ideal gas goes through the following thermal cycle: a reversible isothermal expansion AB at temperature $T_1 = 500$ K; an irreversible expansion BC in which the gas exchanges the quantity of heat $Q_{BC} = -750$ J with a source at temperature $T_3 = 500$ K; a reversible isothermal compression CD at temperature $T_2 = 200$ K; a reversible adiabatic compression DA. The ratios V_B/V_A and V_C/V_D are known, $V_B/V_A = 2$; $V_C/V_D = 2.3$.

- a) Calculate the efficiency of the cycle. [3 points]
- b) Calculate the maximum efficiency achievable by working between sources T_1 and T_2 . [1 point]
- c) Calculate the entropy change of the gas along the four transformations. [2 points]
- d) Calculate the entropy changes of the three sources at temperatures T_1 , T_2 and T_3 . [2 points]



AB: reversible isothermal expansion; BC: irreversible expansion; CD: reversible isothermal compression; DA: reversible adiabatic compression

a) the efficiency of the cycle is given by $e = \frac{W}{Q_{in}}$ $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$W_{AB} = \int P dV = \int \frac{nRT}{V} dV = nRT_1 \ln\left(\frac{V_B}{V_A}\right) = (1\text{mol})(8.31\text{J/mol/K})(500\text{K})\ln 2 = 2880\text{J}$$

$$\text{BC} \left\{ \begin{array}{l} \Delta E_{int.BC} = nC_V(T_2 - T_1) \\ \Delta E_{int.BC} = Q_{BC} - W_{BC} \\ W_{BC} = Q_{BC} - n\frac{5}{2}R(T_2 - T_1) = (-750\text{J}) - (1\text{mol})\frac{5}{2}(8.31\text{J/mol/K})(200\text{K} - 500\text{K}) = 5482\text{J} \end{array} \right\} nC_V(T_2 - T_1) = Q_{BC} - W_{BC} \Rightarrow W_{BC} = Q_{BC} - nC_V(T_2 - T_1) \Rightarrow$$

$$W_{CD} = nRT_2 \ln\left(\frac{V_D}{V_C}\right) = (1\text{mol})(8.31\text{J/mol/K})(200\text{K})\ln\frac{1}{2.3} = -1384\text{J}$$

$$\text{DA} \left\{ \begin{array}{l} \Delta E_{int.DA} = nC_V(T_1 - T_2) \\ \Delta E_{int.DA} = Q_{DA} - W_{DA} = -W_{DA} \end{array} \right\} W_{DA} = -nC_V(T_1 - T_2) = -n\frac{5}{2}R(T_1 - T_2) \\ = -(1\text{mol})\frac{5}{2}(8.31\text{J/mol/K})(500\text{K} - 200\text{K}) = -6232\text{J}$$

The total work of the cycle $W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 2880\text{J} + 5482\text{J} - 1384\text{J} - 6232\text{J} = 746\text{J}$

$$\Delta E_{int.AB} = Q_{AB} - W_{AB} = 0, \Rightarrow Q_{AB} = W_{AB} = 2880\text{J} > 0$$

$$Q_{BC} = -750\text{J} < 0$$

$$\Delta E_{int.CD} = Q_{CD} - W_{CD} = 0, \Rightarrow Q_{CD} = W_{CD} = -1384\text{J} < 0$$

$$Q_{DA} = 0$$

$$e = \frac{W}{Q_{in}} = \frac{W}{Q_{AB}} = \frac{746\text{J}}{2880\text{J}} = 0.26$$

$$\text{b) } e_{max} = e_{ideal} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_2}{T_1} = 1 - \frac{200\text{K}}{500\text{K}} = 0.6$$

c) The entropy change of the gas in a complete cycle $\Delta S_{cyc.} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0\text{J/K}$

$$\text{AB process, } \Delta S_{AB} = \frac{Q_{AB}}{T_1} = \frac{2880\text{J}}{500\text{K}} = 5.76\text{J/K}$$

$$\text{CD process, } \Delta S_{CD} = \frac{Q_{CD}}{T_2} = \frac{-1384\text{J}}{200} = -6.92\text{J/K}$$

$$\text{DA process, } \Delta S_{DA} = 0\text{J/K}$$

$$\text{BC process, } \Delta S_{BC} = \Delta S_{cyc.} - (\Delta S_{AB} + \Delta S_{CD} + \Delta S_{DA}) = 0\text{J/K} - (5.76\text{J/K} - 6.92\text{J/K} + 0\text{J/K}) \\ = 1.16\text{J/K}$$

d)

$$\text{The entropy change of the source at temperatures } T_1, \quad \Delta S_1 = -\Delta S_{AB} = -5.76\text{J/K}$$

$$\text{The entropy change of the source at temperatures } T_2, \quad \Delta S_2 = -\Delta S_{CD} = 6.92\text{J/K}$$

$$\text{The entropy change of the source at temperatures } T_3, \quad \Delta S_3 = -\frac{Q_{BC}}{T_3} = -\frac{-750\text{J}}{500\text{K}} = 1.5\text{J/K}$$

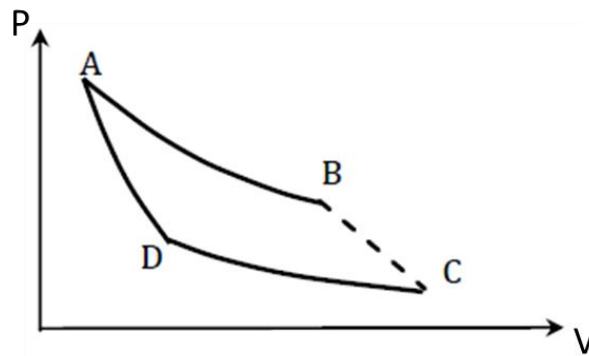
Version 2 and version 4

PHYSICS 1 – A.Y. 2024/2025 Session 1 – JUNE 27, 2025

PROBLEM 3: Thermodynamics [8 points]

One mole of monoatomic ideal gas goes through the following thermal cycle: a reversible isothermal expansion AB at temperature $T_1 = 500$ K, $V_B/V_A = 2$; an irreversible expansion BC in which the gas exchanges the quantity of heat $Q_{BC} = -750$ J with a source at temperature $T_3 = 500$ K; a reversible isothermal compression CD at temperature $T_2 = 200$ K, $V_C/V_D = 2.3$; a reversible adiabatic compression DA.

- e) Calculate the efficiency of the cycle. [3 points]
- f) Calculate the maximum efficiency achievable by working between sources T_1 and T_2 . [1 point]
- g) Calculate the entropy change of the gas along the four transformations. [2 points]
- h) Calculate the entropy changes of the three sources at temperatures T_1 , T_2 and T_3 . [2 points]



AB: reversible isothermal expansion; BC: irreversible expansion; CD: reversible isothermal compression; DA: reversible adiabatic compression

a) the efficiency of the cycle is given by $e = \frac{W}{Q_{in}}$ $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$$W_{AB} = \int P dV = \int \frac{nRT}{V} dV = nRT_1 \ln\left(\frac{V_B}{V_A}\right) = (1\text{mol})(8.31\text{J/mol/K})(500\text{K})\ln 2 = 2880\text{J}$$

$$\text{BC} \left\{ \begin{array}{l} \Delta E_{int.BC} = nC_V(T_2 - T_1) \\ \Delta E_{int.BC} = Q_{BC} - W_{BC} \\ W_{BC} = Q_{BC} - n\frac{3}{2}R(T_2 - T_1) = (-750\text{J}) - (1\text{mol})\frac{3}{2}(8.31\text{J/mol/K})(200\text{K} - 500\text{K}) = 2989.5\text{J} \end{array} \right. \Rightarrow nC_V(T_2 - T_1) = Q_{BC} - W_{BC} \Rightarrow W_{BC} = Q_{BC} - nC_V(T_2 - T_1) \Rightarrow$$

$$W_{CD} = nRT_2 \ln\left(\frac{V_D}{V_C}\right) = (1\text{mol})(8.31\text{J/mol/K})(200\text{K})\ln\frac{1}{2.3} = -1384\text{J}$$

$$\text{DA} \left\{ \begin{array}{l} \Delta E_{int.DA} = nC_V(T_1 - T_2) \\ \Delta E_{int.DA} = Q_{DA} - W_{DA} = -W_{DA} \end{array} \right. \Rightarrow W_{DA} = -nC_V(T_1 - T_2) = -n\frac{3}{2}R(T_1 - T_2) = -(1\text{mol})\frac{3}{2}(8.31\text{J/mol/K})(500\text{K} - 200\text{K}) = -3739.5\text{J}$$

The total work of the cycle $W = W_{AB} + W_{BC} + W_{CD} + W_{DA} = 2880\text{J} + 2989.5\text{J} - 1384\text{J} - 3739.5\text{J} = 746\text{J}$

$$\Delta E_{int.AB} = Q_{AB} - W_{AB} = 0, \Rightarrow Q_{AB} = W_{AB} = 2880\text{J} > 0$$

$$Q_{BC} = -750\text{J} < 0$$

$$\Delta E_{int.CD} = Q_{CD} - W_{CD} = 0, \Rightarrow Q_{CD} = W_{CD} = -1384\text{J} < 0$$

$$Q_{DA} = 0$$

$$e = \frac{W}{Q_{in}} = \frac{W}{Q_{AB}} = \frac{746\text{J}}{2880\text{J}} = 0.26$$

$$\text{b) } e_{max} = e_{ideal} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_2}{T_1} = 1 - \frac{200\text{K}}{500\text{K}} = 0.6$$

c) The entropy change of the gas in a complete cycle $\Delta S_{cyc.} = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0\text{J/K}$

$$\text{AB process, } \Delta S_{AB} = \frac{Q_{AB}}{T_1} = \frac{2880\text{J}}{500\text{K}} = 5.76\text{J/K}$$

$$\text{CD process, } \Delta S_{CD} = \frac{Q_{CD}}{T_2} = \frac{-1384\text{J}}{200} = -6.92\text{J/K}$$

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$$\text{BC process, } \Delta S_{BC} = \Delta S_{cyc.} - (\Delta S_{AB} + \Delta S_{CD} + \Delta S_{DA}) = 0\text{J/K} - (5.76\text{J/K} - 6.92\text{J/K} + 0\text{J/K}) = 1.16\text{J/K}$$

d)

$$\text{The entropy change of the source at temperatures } T_1, \quad \Delta S_1 = -\Delta S_{AB} = -5.76\text{J/K}$$

$$\text{The entropy change of the source at temperatures } T_2, \quad \Delta S_2 = -\Delta S_{CD} = 6.92\text{J/K}$$

$$\text{The entropy change of the source at temperatures } T_3, \quad \Delta S_3 = -\frac{Q_{BC}}{T_3} = -\frac{-750\text{J}}{500\text{K}} = 1.5\text{J/K}$$